SIGPwny x SIGQuantum: Quantum Cryptographic Primitives

Sasha Levinshteyn and Swetha Karthikeyan

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Outline

Impagliazzo's Five Worlds

Modern Cryptography

Classical and Quantum Cryptographic Primitives

Section 1

Impagliazzo's Five Worlds

Obligatory Complexity Speedrun

- A *decision problem* is a question with a yes-or-no-answer related to its inputs.

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- $\mathcal{NP}$ is the class of decision problems that can be verified in polynomial time given a certificate/potential solution.
- So, if $\mathcal{P} \neq \mathcal{NP}$, there exists some decision problem that can be verified but not solved in polynomial time.
- However, if $\mathcal{P} = \mathcal{NP}$, every decision problem that can be efficiently verified can also be efficiently solved.

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- Algorithmica

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- We cannot efficiently solve $\mathcal{NP}$ problems.

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- RSA cannot be trusted.

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- It is easy to come up with hard unsolved problems but hard to come up with solved instances.
- Classical cryptography doesn't exist but we can't solve algorithms quickly either.

Heuristica

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- Classical cryptography is dead.

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Algorithmica

- $\mathcal{P} = \mathcal{NP}$
- Many optimization problems, as well as other kinds of problems, become easy.
- One-way functions do not exist (as they are in $\mathcal{NP}$) and classical cryptography is very, very dead.

Section 2

Modern Cryptography

Secure Cryptography

- Can secure cryptography exist?

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- How do we prove that it does (or doesn't)?

The State of Modern Classical Cryptography

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- They all rely on assumptions about the hardness of specific algebraic problems, called *hardness assumptions.*
  - Factoring
  - Discrete log problem
- What happens if these hardness assumptions are wrong?

Complexity Theoretic Approach

- Ideally, we would like to assume as little as possible while being able to do as much as possible.

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  - Each cryptographic primitive does a specific task in a very specific way.
  - To build an effective cryptographic system from these building blocks, the cryptographic primitives need to be extremely reliable.

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  - Build as many cryptographic primitives from this assumption as possible.
- Determine which primitives imply which other primitives.
  - A primitive P implies another primitive Q if the ability to build P can be manipulated in some way to build Q.

Section 3

Classical and Quantum Cryptographic Primitives

Intro to Quantum

- A *polynomial-time quantum algorithm* is an algorithm that runs in polynomial time on a quantum computer.
- A *quantum state* is essentially a superposition over 'classical states' (bitstrings). We write these as vectors.
- The *inner product* of two quantum states is basically a dot product over the two vectors representing how close the two states are.
- The *no-cloning theorem* states that given a quantum state, you cannot copy it, resulting in two identical quantum states.

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- Candidates: Multiplication and factoring, discrete exponential and logarithm, etc.

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- These are the quantum analog of one-way functions.

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- Pseudorandom generators exist iff one-way functions exist.

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- *Haar random states* are quantum states that are sampled uniformly from the entire state space of a system according to the Haar measure.
- The *Haar measure* is a way to define a uniform probability distribution over the group of unitary operators.
- These are the quantum analog of pseudorandom generators.

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- The probability that a polynomial time adversary A can generate a valid signature for a party without knowing that party's private key should be negligible.

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- These are the quantum analog of digital signatures.

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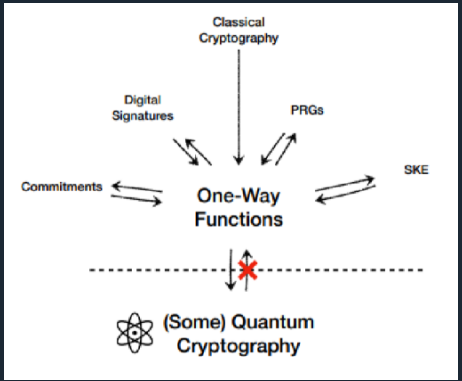
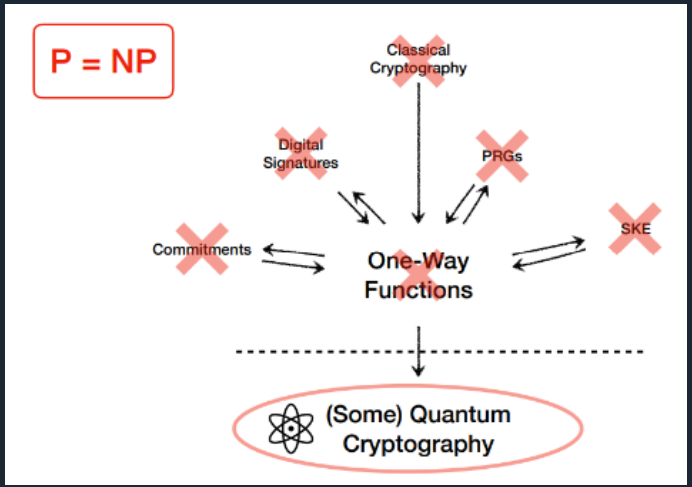


Nightmare Scenario for Cryptography



Minimum Assumptions in Quantum Cryptography

- Quantum cryptography can still work in the nightmare scenario.
- For instance, Quantum Key Distribution (QKD) gets its security from the Laws of Physics.

`https://sattath.github.io/microcrypt-zoo/`

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- Associating classical cryptographic primitives is a huge field of study.
- Researchers are now working on relating quantum cryptographic primitives.
- One-way state generators and EFI pairs are the likely minimum assumptions in quantum cryptography. [KT24]

SIGQuantum

- Join SIGQuantum!!! We meet Tuesdays 6 - 7 pm.
- We run weekly meetings, as well as some projects and Hackathons!
- Find the Discord through the ACM Discord or website.

Bibliography I

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